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IAP5 Rec'd PCT/PTO 28 MAR 2006

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Series of Progressive Ophthalmic Lenses with Low Divergence and  
Rotation of Astigmatism

Our ref.: R 1919 - ro/hf

### Description

This invention relates to a method for producing a progressive ophthalmic lens having at least one progressive surface and a progressive lens having at least one progressive surface.

Progressive ophthalmic lenses, also known as no-line bifocal or multifocal lenses, are known in general. They have a far vision part designed for observing objects at great distances, a near vision part designed for observing objects at closer distances and a progression zone situated between the far vision part and the near vision part. In the progression zone, the effect of the lens increases continuously from the value of the far vision part to the value of the vision part. This increasing effect of the lens is known as addition. A change in curvature of at least one surface of the lens, referred to as the progressive surface, is required to create this progressive effect of the lens. However, a surface astigmatism, which has a negative effect on the optical imaging properties, is unavoidably associated with the progressive effect; this astigmatism is typically equal to zero only along a planar or curved line (principal line) and increases laterally to this line with twice the value of the gradient of the surface refractive power according to the Minkwitz law. Consequently there is impaired vision in the form of distortion and blurring. In dynamic vision, "swimming" and "jumping" of images perceived on the retina are especially annoying and under some circumstances can have a strongly negative effect on the wear properties of the progressive lens.

German Patent DE 43 42 234 C2 describes a progressive lens having a progressive surface, where the maximum value of the gradient of the

average surface refractive value of the progressive surface lies in a part of the principal line situated in the progression zone and the gradient of the cylinder and/or astigmatism of the surface has a value lower than the product obtained by multiplying the addition times a coefficient  $k_{c \max}$  of a constant value over the entire progressive surface of the lens.

The object of the present invention is to provide a progressive ophthalmic lens which has improved wearing properties. Another object of this invention is to provide a method for manufacturing an inventive progressive lens.

This object is achieved according to the present invention by a method for manufacturing a progressive lens having the features characterized in Claim 1 and a progressive lens having the features characterized in Claim 13. Preferred embodiments are characterized in the independent claims.

According to this invention, a method of manufacturing a progressive ophthalmic lens having at least one progressive surface [is proposed], where the lens comprises

- a far vision part designed for seeing at great distances and having a far reference point,
- a near vision part designed for seeing at short distances and having a near reference point,
- a progression zone situated between the far vision part and the near vision part where the effect of the lens increases from the value at the far reference point to the value at the near reference point along a principal line, this increase being by a value known as addition,

whereby a calculation and optimization step [in the production] of the progressive lens is performed so that the absolute value of the rotation  $|\text{rot}\vec{A}|$  and/or the divergence  $|\text{div}\vec{A}|$  of a vectorial astigmatism  $\vec{A}$  is as small as possible, where the absolute value  $|\vec{A}|$  of the vectorial astigmatism  $\vec{A}$  is proportional to the absolute value of an astigmatism in the use position of the progressive lens or a surface astigmatism of the

at least one progressive surface of the progressive lens, and the direction of the vectorial astigmatism  $\vec{A}$  is proportional to the cylinder axis of an astigmatism in the use position of the progressive lens or a surface astigmatism of the at least one progressive surface of the progressive lens.

The far vision part is usually situated in the upper part of the lens in the use position and the near vision part is located in the lower part of the lens. The far vision part is usually designed for seeing at an infinite distance and the near vision part is designed for reading in particular. In the case of ophthalmic lenses for special applications, e.g., lenses for pilot's glasses or glasses for working at a computer monitor, the far vision part and the near vision part may be arranged differently and designed for different distances. Furthermore, there may also be multiple near vision parts and far vision parts as well as progression zones.

The progressive lens may have one progressive surface (front or back surface) or two progressive surfaces. The progressive surface may be the side of the lens facing the eyes, i.e., the back side or the object side, i.e., the front side of the lens, but the opposing surface usually has a spherical or toric surface, which does not contribute to the surface variation of the astigmatism of the lens. In the case of a lens having two progressive surfaces, the derivations of the vectorial astigmatism  $|\text{rot}\vec{A}|$  and  $|\text{div}\vec{A}|$ , which are to be kept small, are formed due to the combined effect of the front surface and the back surface (i.e., the surface facing the eyes and the object surface). The value of the addition is calculated as the difference in the use values of the refractive force at the far reference point and at the near reference point.

According to this invention, a progressive lens whose properties are especially advantageous, i.e., as small as possible, with respect to the spatial variations in the astigmatism including its directional information is made available. According to this invention, it is recognized that such directional variations are the cause of the change in visual impression known as the swimming or jumping that occurs with movements of the head or line of sight.

In particular, it has been recognized that the change or variation in the cylinder axis of the astigmatism is not only a secondary parameter but on the contrary is an important parameter. In comparison with that, lenses corresponding to the state of the art have so far never been calculated taking into account the spatial variation in their surface astigmatism including its direction information.

Of the properties of the surfaces of progressive lenses, so far in the literature at most the second derivatives of the depths of camber have usually been discussed. In some cases such as in DE 4 342 234 C2, gradients of the surface refractive value  $\nabla D(x, y)$  or the surface astigmatism  $\nabla A(x, y)$ , i.e., third derivations of the depths of camber of the surface have been discussed. The aforementioned patent refers in several places to the relevance of these gradients for dynamic vision and thus for the site conditions associated with movement of the head or line of site or combinations thereof.

However, astigmatism is always interpreted here only as a two-dimensional scalar function which is usually defined as being proportional to the difference  $|1/r_1 - 1/r_2|$  of the main radii of curvature. The conventional approach disregards the cylinder axis of the astigmatism and concentrates only on its absolute value or as in DE 4 342 234 C2, concentrates on the gradient of the absolute value of the surface astigmatism.

The present invention is based on the new finding that with a spatial change in a quantity which also has an cylinder axis as is the case with astigmatism, it is not only the change in the absolute amount that is of interest but also the change in its directional information. However, any information regarding the change in cylinder axis of the astigmatism which is critical for the optical correction is lost when the vector field of the astigmatism is reduced to its absolute value.

The role played by the cylinder axis of astigmatism can be illustrated on the basis of a determination of the refraction, for example. The determination of the cylinder axis of the astigmatism is performed here in sequence before the determination of the absolute value of the astigmatism. The reason for this is that in a

determination of the cylinder axis, e.g., by the cross-cylinder method, a cylinder whose absolute value is estimated is modified with regard to its cylinder axis with the help of the cross-cylinder until it has been defined optimally. This is followed by a final determination of the absolute value of the cylinder. An attempt to determine the absolute value of the cylinder even before defining the axis would result in skewed cylinders in the course of the determination of absolute value. This would mean that a resulting cylinder whose absolute value depends on the size of the correction cylinder and the size of the axial error would be obtained. One would thus have two constantly changing parameters which would have to be repeatedly corrected again in alternation. However, if the cylinder axis of the astigmatism is already known, then the correction cylinder is always changed by the full amount, keeping the cross cylinder in reserve. Finally, if the axis of the correction cylinder has been determined correctly, it will coincide with that of the refraction deficit and the resulting cylindrical effect will be zero, as described in the book by H. Diepes, *Refraktionsbestimmung* [Determining Refraction], Verlag Heinz Postenrieder, Pforzheim (1972), page 322, for example.

In addition to having a negative effect on vision, any astigmatism also results in distortion, as already indicated above. The subjective perception of the resulting distortion of the retinal images also depends on the direction of the cylinder axis as described, for example, by J. Reiner in *Auge und Brille* [Eye and Lens], 4<sup>th</sup> edition, Ferdinand Enke Verlag Stuttgart (1987), page 27. In the case of an astigmatism according to the rule or against the rule, the result is distortion, which does not usually cause much interference for the wearer of the lens, because the distortion is only in the vertical or horizontal direction. Distortion of this type is often caused by viewing from a lateral position so it is not unknown. Problems occur for the users of ophthalmic lenses mainly in astigmatism with an oblique cylinder axis because the image of vertical and horizontal object elements on the retina is skewed. Distortion of this type is much less tolerable because vertical and horizontal lines no longer appear at right angles in space but instead form angles deviating from 90° (greater than or smaller than 90°). This results in fusion problems,

which make it difficult to become adjusted to lenses. According to this invention, it has been found that such stresses are to be expected for sensory fusion when the cylinder axis of the astigmatism changes in the course of lateral eye movements. If astigmatic effects occur, which remain the same in terms of absolute value but now have an oblique cylinder axis which also changes depending on the point of viewing through the lens, this can result in intolerance of the lens in the peripheral area.

To be able to describe spatial variations in astigmatism, first a suitable vectorial astigmatism is defined. The spatial distribution of this vectorial astigmatism defines a vector field accordingly. When the change in absolute value and direction (corresponding to the absolute value and the cylinder axis of the astigmatism) of this vectorial astigmatism is integrated into the calculation, i.e., optimization of a lens and in particular a progressive lens, this yields new aspects which lead to improved imaging properties and thus improved wearing properties, as will be explained below. It is especially expedient for describing the derivations of the vectorial astigmatism  $\vec{A}$  to consider its rotation and divergence, i.e., the vector operators  $\text{rot}\vec{A} = \vec{\nabla} \times \vec{A}$  and  $\text{div}\vec{A} = \vec{\nabla} \cdot \vec{A}$ .

Although the present invention refers to both the consideration of the surface astigmatism of a progressive lens as well as astigmatism in the use position, surface astigmatism of the progressive surface is considered below as an example. Therefore, a detailed definition of a vectorial surface astigmatism is given below, and then a brief similar definition of a vectorial astigmatism in the use position is also given.

#### **Definition of the vectorial surface astigmatism $\vec{A}$**

The depth of camber of a progressive surface is given by the function

$$z = f(x, y)$$

where the coordinates  $(x, y)$  are in the plane of the projection. The depth of camber is understood to refer to the distance of a point with coordinates  $(x, y)$  from the tangential plane of the surface vertex.

The first basic form

$$g(x, y) = \begin{pmatrix} E & F \\ F & G \end{pmatrix} = \begin{pmatrix} 1 + f_x^2 & f_x f_y \\ f_x f_y & 1 + f_y^2 \end{pmatrix}$$

and the second basic form

$$l(x, y) = \begin{pmatrix} K & L \\ L & M \end{pmatrix} = \frac{-1}{\sqrt{1 + f_x^2 + f_y^2}} \begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{pmatrix}$$

are needed to calculate the main curvatures, where the derivations are abbreviated by  $f_x = \partial_x f(x, y)$ ,  $f_y = \partial_y f(x, y)$ ,  $f_{xx} = \partial_x^2 f(x, y)$ ,  $f_{yy} = \partial_y^2 f(x, y)$ ,  $f_{xy} = \partial_x \partial_y f(x, y)$ . The main curvatures  $k_1$  and  $k_2$  and the main curvature directions are solutions to the general eigenvalue problem

$$\begin{pmatrix} K & L \\ L & M \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} - \begin{pmatrix} E & F \\ F & G \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} = 0. \quad (1)$$

The eigenvectors  $(x_1, y_1)$  and  $(x_2, y_2)$  calculated in this way are the projections of the vectors in the tangential plane of the lens surface and indicate the main directions of curvature there.

To calculate the main direction of curvature, the eigenvectors  $(x_1, y_1)$  and  $(x_2, y_2)$  must be transformed to the system of the tangential plane after their calculation. This transformation can best be determined by formulating the entire eigenvalue problem itself in the tangential plane. This is done by bringing it to the form

$$\begin{pmatrix} K & L \\ L & M \end{pmatrix}_{\text{eff}} \begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix} - \begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} = 0. \quad (2)$$

because the eigenvectors  $(u_1, v_1)$  and  $(u_2, v_2)$  must lie in the tangential plane since the first basic form in this presentation is the unit matrix.

It seems simplest to multiply the original problem (1) simply from the left times  $\begin{pmatrix} E & F \\ F & G \end{pmatrix}^{-1}$ . However, this would lead to an asymmetrical matrix

$\begin{pmatrix} K & L \\ L & M \end{pmatrix}_{\text{eff}}$  whose eigenvectors would no longer be orthogonal because the  $(u, v)$  coordinate system would then be skewed. To retain symmetry, the first basic form  $g$  must first be transformed to a diagonal form:

$$W^T g W = g_d \leftrightarrow g = W g_d W^T = (W g_d^{1/2}) (W g_d^{1/2})^T$$

where  $g_d$  is a diagonal matrix with the eigenvalues of  $g$  and  $W$  is a matrix whose columns have the eigenvectors of  $g$ . The transformation  $(W g_d^{1/2})^T$  transforms a vector  $(x, y)$  to the tangential plane by means of

$$\begin{pmatrix} u \\ v \end{pmatrix} = (W g_d^{1/2})^T \begin{pmatrix} x \\ y \end{pmatrix}$$

because the length of a vector in the  $(u, v)$  plane is given by

$$\begin{aligned} (u \ v) \begin{pmatrix} u \\ v \end{pmatrix} &= (x \ y) (W g_d^{1/2}) (W g_d^{1/2})^T \begin{pmatrix} x \\ y \end{pmatrix} \\ &= (x \ y) g \begin{pmatrix} x \\ y \end{pmatrix}. \end{aligned}$$

With each transformation  $(W g_d^{1/2})^T$  to the  $(u, v)$  plane,  $(W g_d^{1/2} R)^T$  is also such a transformation when  $R = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix}$  is a rotation within the  $(u, v)$  plane. It is practical now to setup the rotation  $R$  in such a



way that the  $u$  axis coincides with the intersection formed by the tangential plane with the horizontal plane. This can be achieved by an angle  $\phi$  for which the lower left matrix element of  $(Wg_d^{1/2}R)^T$  disappears. As can be confirmed by calculation, this is the case for  $\phi_0 = \arctan \frac{f_y}{f_x} \sqrt{1 + f_x^2 + f_y^2}$  and the resulting transformation is then given by

$$T = (Wg_d^{1/2}R(\phi_0))^T = \frac{\text{Sign}(f_x)}{\sqrt{(1 + f_y^2)(1 + f_x^2 + f_y^2)}} \begin{pmatrix} (1 + f_y^2) & 0 \\ -f_x f_y & \sqrt{1 + f_x^2 + f_y^2} \end{pmatrix} \text{ and}$$

$$T^{-1} = \frac{\text{Sign}(f_x)}{\sqrt{1 + f_y^2}} \begin{pmatrix} \sqrt{1 + f_x^2 + f_y^2} & 0 \\ f_x f_y & 1 + f_y^2 \end{pmatrix}$$

The eigenvalue problem can now be written by inserting  $g = T^T T$  in the form:

$$\begin{pmatrix} K & L \\ L & M \end{pmatrix} \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} - T^T T \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} = 0 \leftrightarrow$$

$$(T^{-1})^T \begin{pmatrix} K & L \\ L & M \end{pmatrix} T^{-1} \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} - T \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} = 0 \leftrightarrow$$

$$\begin{pmatrix} K & L \\ L & M \end{pmatrix}_{\text{eff}} \begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix} - \begin{pmatrix} u_1 & u_2 \\ v_1 & v_2 \end{pmatrix} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} = 0.$$

The eigenvalue problem transformed in this way to the tangential plane contains a symmetrical representation of the second basic form:

$$\begin{pmatrix} K & L \\ L & M \end{pmatrix}_{\text{eff}} = (T^{-1})^T \begin{pmatrix} K & L \\ L & M \end{pmatrix} T^{-1} =$$

$$\frac{1}{(1 + f_y^2)\sqrt{1 + f_x^2 + f_y^2}} \left( \begin{array}{c} \frac{-f_x(1 + f_y^2) + f_x f_y(-f_x f_y^2 + 2f_{xy}(1 + f_y^2))}{1 + f_x^2 + f_y^2} \\ \frac{f_x f_y^2 - f_{xy}(1 + f_y^2)}{\sqrt{1 + f_x^2 + f_y^2}} \end{array} \quad \begin{array}{c} \frac{f_x f_y^2 - f_{xy}(1 + f_y^2)}{\sqrt{1 + f_x^2 + f_y^2}} \\ -f_y \end{array} \right)$$

and thus yields the main curvatures  $k_1$  and  $k_2$  and the respective main directions of curvature  $(u_1, v_1)$  and  $(u_2, v_2)$ .

For definition of a vectorial astigmatism, i.e., a vector field for the astigmatism, the absolute value of which is given by the quantity  $|k_1 - k_2|$ , it is advantageous to double the angle; this is usually also done to check the cylinder axis of the TABO scheme in polar coordinates as proposed in the publication WO 01/81979 for example. If the direction of the first cylinder axis is given approximately by the angle  $\psi = \arctan v_1/u_1$ , then a vectorial astigmatism can be defined as follows:

$$\bar{A} = (n - 1) |k_2 - k_1| \begin{pmatrix} \cos 2\psi \\ \sin 2\psi \end{pmatrix} =$$

$$\frac{n - 1}{(1 + f_y^2)(1 + f_x^2 + f_y^2)^{3/2}} \left( \begin{array}{c} -2f_x f_{xy} f_y (1 + f_y^2) + f_{xx}(1 + f_y^2)^2 + f_x^2(-1 + f_y^2)f_{yy} - (1 + f_y^2)f_{yy} \\ 2\sqrt{1 + f_x^2 + f_y^2}(f_{xy} + f_{xy}f_y^2 - f_x f_y f_{yy}) \end{array} \right)$$

(3),

where  $n$  is the refractive index.

It should be pointed out here that the second cylinder axis does not lead to a new vectorial astigmatism but instead causes only a global plus or minus sign: the second cylinder axis is perpendicular to the first and the right angle is expanded to 180° by doubling the angle.

By analogy with vectorial surface astigmatism, an astigmatism in use position is defined vectorial as  $|k_2 - k_1|(\cos 2\psi, \sin 2\psi)$  where  $k_{1,2}$  are

not the eigenvalues of the second basic form of the refractive surface, however, but instead the emergent wave front. The quantity  $(n - 1)$  which depends on the refractive index is contained in the eigenvalues  $k_{1,2}$ —in contrast with the situation with the surface astigmatism in equation (3)—and therefore need not be multiplied as an additional factor. The angle  $\psi$  is intended in the tangential plane at the wave front and has as the reference direction the line of intersection of this plane with the horizontal plane. For perpendicular incidence of a wave front from the infinite, the factor given in equation (3) is obtained for the vectorial astigmatism in the use position; otherwise deviating values are obtained.

In the following discussion, the astigmatism  $\bar{A}$  is understood to be the vectorial surface astigmatism defined according to equation (3).

A first approximation of the vectorial astigmatism is obtained by assuming that the first derivations  $f_x$ ,  $f_y$  of the depth of camber are small in comparison with 1, i.e.,  $1 + f_x^2 \approx 1$ ,  $1 + f_y^2 \approx 1$ , etc. For the vectorial astigmatism it then holds that:

$$\bar{A} = (n - 1) \begin{pmatrix} f_{xx} - f_{yy} \\ 2f_{xy} \end{pmatrix} \quad (4)$$

It follows from equation (4) that the vectorial astigmatism is an expression of the second derivations of the depth of camber.

On the whole, this yields four independent derivations of the vector field  $\bar{A}$ , namely  $\frac{\partial A_x}{\partial x}$ ,  $\frac{\partial A_x}{\partial y}$ ,  $\frac{\partial A_y}{\partial x}$  and  $\frac{\partial A_y}{\partial y}$ . They are all of the same order of magnitude. Although they are all of interest, the following discussion is limited to two especially customary combinations of these four derivations of the vectorial astigmatism  $\bar{A}$ , namely its divergence

$$\text{div } \bar{A} = \bar{\nabla} \cdot \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} \quad (5)$$

and the z component of its rotation

$$(\text{rot} \bar{A})_z = (\bar{V} \times \bar{A})_z = \frac{\partial A_y}{\partial x} + \frac{\partial A_x}{\partial y}. \quad (6)$$

For reasons of simplicity,  $\text{rot} \bar{A}$  is written below instead of  $(\text{rot} \bar{A})_z$  because  $\text{rot} \bar{A} = (0, 0, (\text{rot} \bar{A})_z)$ .

The exact formulas for  $\text{rot} \bar{A}$  and  $\text{div} \bar{A}$  derived from equation (3) for the vectorial astigmatism  $\bar{A}$  are not given here for reasons of space but instead the exact results obtained from them are plotted graphically below. When the approximation obtained according to equation (3) for the vectorial astigmatism is inserted to equations (5) and (6), this yields the following approximations for the divergence and rotation of the vectorial astigmatism:

$$\text{div} \bar{A} \propto f_{xxx} + f_{xyy} \text{ and } (\text{rot} \bar{A})_z \propto f_{yyy} + f_{xxy} \quad (7)$$

It can be seen from equation (7) that the rotation and divergence of the vectorial astigmatism  $\bar{A}$  characterize the derivations of the depth of camber. There are a total of four independent third derivations of the depth of camber, namely  $f_{xxx}$ ,  $f_{xxy}$ ,  $f_{xyy}$ ,  $f_{yyy}$ .

Ophthalmic lenses manufactured by the inventive method have definitely improved wear properties in comparison with ophthalmic lenses produced by conventional methods without optimization with regard to the absolute value as well as the direction of astigmatism of the ophthalmic lens. It should be pointed out that the present patent application refers to the astigmatism inherent in a progressive lens which does not serve to correct the natural astigmatism of the eye (the latter is constant over the entire lens anyway and therefore has negligible derivations).

The calculation and optimization step is preferably performed in such a way that a global maximum of the absolute value  $|\text{div} \bar{A}|$  of the divergence of the vectorial astigmatism  $\bar{A}$  is outside of the zone of

good vision of the lens, in which the absolute value of the vectorial astigmatism  $|\bar{A}|$  is less than 0.6 dpt, preferably less than or equal to 0.5 dpt, and is preferably located in the peripheral area of the lens.

This achieves the result that the interfering maximums and minimums of  $\text{div}\bar{A}$  are shifted into a range which is not often used for vision purposes and therefore the wearing properties of the lens are improved. An astigmatism with a value of more than 0.5 dpt leads to an image that is perceived as blurred on the retina.

The calculation and optimization step is also preferably performed in such a way that the x coordinate of the position of the global maximum of the absolute value  $|\text{div}\bar{A}|$  of the divergence of the vectorial astigmatism  $\bar{A}$  is greater than 6.0 mm and the y coordinate is less than -8.5 mm, where x is the horizontal axis and y is the vertical axis in the use position, and the zero point  $x = 0$ ,  $y = 0$  is 4 mm below the centering point of the lens.

The horizontal axis x is therefore parallel to the direction which is defined by two permanent marks on the rimless lens which are a distance of 17 mm laterally from the principal line. The vertical axis y is perpendicular to the horizontal direction. The zero point  $x = 0$ ,  $y = 0$  is 4 mm below the centering point of the lens and corresponds to the center of the lens in the case of non-predecentered lenses. For most lenses this is also the prism reference point, i.e., the point at which the prismatic effect of the lens is to be determined. The centering point is 4 mm above the midpoint of the lens and is prestamped on the lens in the form of a cross. In observation of the surface astigmatism of the progressive surface, x and y are in a tangential plane passing through the vertex of the progressive surface.

Within the range  $y \geq -8$  mm, there are no mentionable extremes at all of the divergence  $|\text{div}\vec{A}|$  of the vectorial astigmatism  $\vec{A}$ . This feature is satisfied for all additions.

An extreme of  $\text{div}\vec{A}$  is considered "mentionable" if its absolute value is more than (0.1/mm) times the addition.

Furthermore, the calculation and optimization step is preferably performed so that all extremes of the amount  $|\text{div}\vec{A}|$  of the divergence of the vectorial astigmatism  $\vec{A}$  exceeding the value of (0.1/mm) times the addition are outside the range  $y \geq -9$  mm of the lens for all progressive surfaces with addition  $\geq 2.0$  dpt. This feature is satisfied for all basic curves.

In addition, it is also preferable if the calculation and optimization step is performed so that the absolute value  $|\text{rot}\vec{A}|$  of the rotation of the vectorial astigmatism  $\vec{A}$  in the near vision part and/or in the far part does not increase beyond a maximum value of  $|\text{rot}\vec{A}|_{\max} \approx 0.25$  addition/dpt·dpt/mm.

The calculation and optimization step is especially preferably performed so that the absolute value  $|\text{rot}\vec{A}|$  of the rotation of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = -14$  mm does not exceed a maximum value of  $|\text{rot}\vec{A}|_{\max} \approx 0.115$  addition/dpt·dpt/mm, preferably  $|\text{rot}\vec{A}|_{\max} \approx 0.08$  addition/dpt·dpt/mm.

Furthermore, the calculation and optimization step is preferably performed so that the absolute value  $|\text{rot}\vec{A}|$  of the rotation of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = +6$  mm does not exceed a maximum value of  $|\text{rot}\vec{A}|_{\max} \approx 0.115$  addition/dpt·dpt/mm, preferably  $|\text{rot}\vec{A}|_{\max} \approx 0.06$  addition/dpt·dpt/mm.

The calculation and optimization step is preferably performed so that in the far vision part between  $y = 3 \text{ mm}$  and  $y = 5 \text{ mm}$  there is at least one horizontal section  $y = \text{const}$  along which the value  $|\text{rot}\vec{A}|$  of the rotation of the vectorial astigmatism  $\vec{A}$  increases monotonically from the principal line outward to a coordinate of  $|x| = 16 \text{ mm}$ .

Furthermore, it is preferable for the calculation and optimization step to be performed so that the divergence  $\text{div}\vec{A}$  of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = 0 \text{ mm}$  does not increase beyond a maximum value of  $(\text{div}\vec{A})_{\text{max}} \approx (0.11 \text{ addition/dpt} + 0.03) \text{ dpt/mm}$ , preferably  $(\text{div}\vec{A})_{\text{max}} \approx (0.08 \text{ addition/dpt} + 0.03) \text{ dpt/mm}$ .

The calculation and optimization step is preferably performed so that the divergence  $\text{div}\vec{A}$  of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = 0 \text{ mm}$  does not fall below a minimum value of  $(\text{div}\vec{A})_{\text{min}} \approx (-0.07 \text{ addition/dpt} - 0.11) \text{ dpt/mm}$ , preferably  $(\text{div}\vec{A})_{\text{min}} \approx (-0.05 \text{ addition/dpt} - 0.08) \text{ dpt/mm}$ .

A method for manufacturing a progressive lens that is especially preferred is one in which the calculation and optimization step is performed so that the divergence  $\text{div}\vec{A}$  of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = -14 \text{ mm}$  does not increase beyond a maximum value of  $(\text{div}\vec{A})_{\text{max}} \approx (0.12 \text{ addition/dpt} + 0.06) \text{ dpt/mm}$ .

In addition it is preferable if the calculation and optimization step is performed so that the divergence  $\text{div}\vec{A}$  of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = -14 \text{ mm}$  does not drop below a minimum value of  $(\text{div}\vec{A})_{\text{min}} \approx (-0.13 \text{ addition/dpt} - 0.05) \text{ dpt/mm}$ .

Furthermore, according to this invention, a progressive lens having at least one progressive surface is provided, whereby the lens has at least:

- one far vision part designed for seeing at great distances and having a far reference point;
- one near vision part designed for seeing at short distances and having a near reference point and
- a progression zone situated between the far and near vision parts where the effect of the lens increases along a principal line by a value (referred to as "addition" or "add power") from the value at the far reference point to the value at the near reference point,

whereby

- the global maximum of the absolute value  $|\text{div}\bar{A}|$  of the divergence of a vectorial astigmatism  $\bar{A}$  is situated outside of the zone of good vision of the lens in which the absolute value of vectorial astigmatism  $|\bar{A}|$  is less than 0.6 dpt, preferably less than or equal to 0.5 dpt and is preferably situated in the peripheral area of the lens and/or
- the absolute value  $|\text{rot}\bar{A}|$  of the rotation of the vectorial astigmatism  $\bar{A}$  in the near vision part and/or in the far vision part does not exceed a maximum value of  $|\text{rot}\bar{A}|_{\text{max}} \approx 0.25 \text{ addition/dpt}\cdot\text{dpt}/\text{mm}$ , and

whereby the absolute value  $|\bar{A}|$  of the vectorial astigmatism  $\bar{A}$  is proportional to the absolute value of an astigmatism in the use position of the progressive lens or a surface astigmatism of the at least one progressive surface of the progressive lens, and the direction of the vectorial astigmatism  $\bar{A}$  is proportional to the cylinder axis of an astigmatism in the use position of the progressive lens or a surface astigmatism of the at least one progressive surface of the progressive lens.

With regard to the important features and definitions, reference is made to the detailed description of the method for producing a lens as described above.



The  $x$  coordinate of the position of the global maximum of the absolute value  $|\text{div}\bar{A}|$  of the divergence of the vectorial astigmatism  $\bar{A}$  is preferably greater than 6.0 mm and the  $y$  coordinate is preferably less than -8.5 mm, where  $x$  denotes the horizontal axis and  $y$  denotes the vertical axis in the used position, and the zero point  $x = 0$ ,  $y = 0$  is situated 4 mm beneath the centering point of the lens.

For all progressive surfaces with addition  $\geq 2.0$  dpt, it is also preferable for all extremes of the absolute value  $|\text{div}\bar{A}|$  of the divergence of the vectorial astigmatism  $\bar{A}$  exceeding the value  $(0.1/\text{mm})$  times the addition to be outside of the range  $y \geq -9$  mm of the lens.

The absolute value  $|\text{rot}\bar{A}|$  of the vectorial astigmatism  $\bar{A}$  in the horizontal section at  $y = -14$  mm especially preferably does not increase beyond a maximum value of  $|\text{rot}\bar{A}|_{\max} \approx 0.115$  addition/dpt·dpt/mm, preferably  $|\text{rot}\bar{A}|_{\max} \approx 0.08$  addition/dpt·dpt/mm.

The absolute value  $|\text{rot}\bar{A}|$  of the rotation of the vectorial astigmatism  $\bar{A}$  in the horizontal section at  $y = +6$  mm preferably does not increase beyond a maximum value of  $|\text{rot}\bar{A}|_{\max} \approx 0.115$  addition/dpt·dpt/mm, preferably  $|\text{rot}\bar{A}|_{\max} \approx 0.06$  addition/dpt·dpt/mm.

It is also preferable if in the far vision part between  $y = 3$  mm and  $y = 5$  mm there is at least one horizontal section  $y = \text{const}$  along which the absolute value  $|\text{rot}\bar{A}|$  of the rotation of the vectorial astigmatism  $\bar{A}$  increases monotonically from the principal line outward to a coordinate of  $|x| = 16$  mm.

It is also preferable if the divergence  $\text{div}\bar{A}$  of the vectorial astigmatism  $\bar{A}$  in the horizontal section at  $y = 0$  mm does not exceed a maximum value of

$(\text{div} \vec{A})_{\max} \approx (0.11 \text{ addition/dpt} + 0.03) \text{ dpt/mm}$ , preferably

$(\text{div} \vec{A})_{\max} \approx (0.08 \text{ addition/dpt} + 0.03) \text{ dpt/mm}$ .

The divergence  $\text{div} \vec{A}$  of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = 0 \text{ mm}$  preferably does not drop below a minimum value of

$(\text{div} \vec{A})_{\min} \approx (-0.07 \text{ addition/dpt} - 0.11) \text{ dpt/mm}$ , preferably

$(\text{div} \vec{A})_{\min} \approx (-0.05 \text{ addition/dpt} - 0.08) \text{ dpt/mm}$ .

It is also preferable if the divergence  $\text{div} \vec{A}$  of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = -14 \text{ mm}$  does not exceed a maximum value of  $(\text{div} \vec{A})_{\max} \approx (0.12 \text{ addition/dpt} + 0.06) \text{ dpt/mm}$ .

It is especially preferable if the divergence  $\text{div} \vec{A}$  of the vectorial astigmatism  $\vec{A}$  in the horizontal section at  $y = -14 \text{ mm}$  does not drop below a minimum value of  $(\text{div} \vec{A})_{\min} \approx (-0.13 \text{ addition/dpt} - 0.05) \text{ dpt/mm}$ .

This invention is explained in greater detail below on the basis of the accompanying drawings of preferred embodiments as examples.

FIG 1A shows the plot of the surface refractive value of the front surface of a lens according to the state of the art in dpt as a function of the coordinates  $(x, y)$  in mm;

FIG 1B shows the plot of the surface refractive value of the front surface of a lens according to exemplary embodiment 1 in dpt as a function of the coordinates  $(x, y)$  in mm;

FIG 1C shows the plot of the surface refractive value of the front surface of a lens according to exemplary embodiment 2 in dpt as a function of the coordinates  $(x, y)$  in mm;

- FIG 2A shows the plot of the absolute value of the surface astigmatism of the front surface of a lens according to the state of the art in dpt as a function of the coordinates  $(x, y)$  in mm;
- FIG 2B shows the plot of the absolute value of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt as a function of the coordinates  $(x, y)$  in mm;
- FIG 2C shows the plot of the absolute value of the surface astigmatism of the front surface of a lens according to exemplary embodiment 2 in dpt as a function of the coordinates  $(x, y)$  in mm;
- FIG 3A shows the plot of the gradient of absolute value of the surface astigmatism of the front surface of a lens according to the state of the art in dpt as a function of the coordinates  $(x, y)$  in mm;
- FIG 3B shows the plot of the gradient of absolute value of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt as a function of the coordinates  $(x, y)$  in mm;
- FIG 3C shows the plot of the gradient of absolute value of the surface astigmatism of the front surface of a lens according to exemplary embodiment 2 in dpt/mm as a function of the coordinates  $(x, y)$  in mm;
- FIG 4A shows the vector field of the surface astigmatism of the front surface of a lens according to the state of the art as a function of the coordinates  $(x, y)$  in mm;
- FIG 4B shows the vector field of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 as a function of the coordinates  $(x, y)$  in mm;

- FIG 4C shows the vector field of the surface astigmatism of the front surface of a lens according to exemplary embodiment 2 as a function of the coordinates  $(x, y)$  in mm;
- FIG 5A shows the rotation  $\text{rot}\vec{A}$  of the surface astigmatism of the front surface of a lens according to the state of the art in dpt/mm as a function of the coordinates  $(x, y)$  in mm;
- FIG 5B shows the rotation  $\text{rot}\vec{A}$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt/mm as a function of the coordinates  $(x, y)$  in mm;
- FIG 5C shows the rotation  $\text{rot}\vec{A}$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 2 in dpt/mm as a function of the coordinates  $(x, y)$  in mm;
- FIG 6A shows the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according to the state of the art in dpt/mm as a function of the coordinates  $(x, y)$  in mm;
- FIG 6B shows the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt/mm as a function of the coordinates  $(x, y)$  in mm;
- FIG 6C shows the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 2 in dpt/mm as a function of the coordinates  $(x, y)$  in mm;
- FIG 7A shows the plot of the absolute value of the rotation  $|\text{rot}\vec{A}|$  of the surface astigmatism of the front surface of a lens according to the state of the art in dpt/mm at a horizontal section  $y = 3$  mm;
- FIG 7B shows the plot of the absolute value of the rotation  $|\text{rot}\vec{A}|$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt/mm at a horizontal section  $y = 4.8$  mm;

- FIG 7C shows the plot of the absolute value of the rotation  $|\text{rot}\tilde{A}|$  of the surface astigmatism of the front surface of a lens according exemplary embodiment 2 in dpt/mm at a horizontal section  $y = 3 \text{ mm}$ ;
- FIG 7D shows the plot of the absolute value of the rotation  $|\text{rot}\tilde{A}|$  of the surface astigmatism of the front surface of a series of lenses according to exemplary embodiment 2 having additions of 2.0, 2.5 and 3.0 dpt in dpt/mm at a horizontal section  $y = 3 \text{ mm}$ ;
- FIG 8A shows the plot of the absolute value of the rotation  $|\text{rot}\tilde{A}|$  of the surface astigmatism of the front surface of a lens according to the state of the art in dpt/mm at a horizontal section  $y = 6 \text{ mm}$ ;
- FIG 8B shows the plot of the absolute value of the rotation  $|\text{rot}\tilde{A}|$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt/mm at a horizontal section  $y = 6 \text{ mm}$ ;
- FIG 8C shows the plot of the absolute value of the rotation  $|\text{rot}\tilde{A}|$  of the surface astigmatism of the front surface of a lens according exemplary embodiment 2 in dpt/mm at a horizontal section  $y = 6 \text{ mm}$ ;
- FIG 8D shows the plot of the absolute value of the rotation  $|\text{rot}\tilde{A}|$  of the surface astigmatism of the front surface of a series of lenses according to exemplary embodiment 1 having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = 6 \text{ mm}$ ;
- FIG 8E shows the plot of the absolute value of the rotation  $|\text{rot}\tilde{A}|$  of the surface astigmatism of the front surface of a series of lenses according to exemplary embodiment 2 having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = 6 \text{ mm}$ ;

- FIG 9A shows the plot of the absolute value of the rotation  $|\text{rot}\vec{A}|$  of the surface astigmatism of the front surface of a lens according to the state of the art in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 9B shows the plot of the absolute value of the rotation  $|\text{rot}\vec{A}|$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 9C shows the plot of the absolute value of the rotation  $|\text{rot}\vec{A}|$  of the surface astigmatism of the front surface of a lens according exemplary embodiment 2 in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 9D shows the plot of the absolute value of the rotation  $|\text{rot}\vec{A}|$  of the surface astigmatism of the front surface of a series of lenses according to exemplary embodiment 1 having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 9E shows the plot of the absolute value of the rotation  $|\text{rot}\vec{A}|$  of the surface astigmatism of the front surface of a series of lenses according to exemplary embodiment 2 having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 10A shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according to the state of the art in dpt/mm at a horizontal section  $y = 0$  mm;
- FIG 10B shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt/mm at a horizontal section  $y = 0$  mm;

- FIG 10C shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according exemplary embodiment 2 in dpt/mm at a horizontal section  $y = 0$  mm;
- FIG 10D shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a series of lenses according to exemplary embodiment 1 having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = 0$  mm;
- FIG 10E shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = 0$  mm;
- FIG 11A shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according to the state of the art in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 11B shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a lens according to exemplary embodiment 1 in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 11C shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface in dpt/mm at a horizontal section  $y = -14$  mm according to exemplary embodiment 2;
- FIG 11D shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a series of lenses according to exemplary embodiment 1 having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = -14$  mm;
- FIG 11E shows the plot of the divergence  $\text{div}\vec{A}$  of the surface astigmatism of the front surface of a series of lenses

according to exemplary embodiment 2 having additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt in dpt/mm at a horizontal section  $y = -14$  mm.

The features of this invention are explained in greater detail below on the basis of two exemplary embodiments (exemplary embodiment 1 and exemplary embodiment 2). Furthermore a comparison is made with a traditional ophthalmic lens according to the state of the art.

All comparable lenses (the lenses of the two exemplary embodiments and the state of the art) have a spheric/toric back surface (ocular surface) and a progressive front surface (object surface). In this case, the rotational and divergence properties of the astigmatism can be explained on the basis of the properties of the front surface alone. The progressive surface of the lenses according to the two inventive embodiments each have a basic curve of 4 dpt and addition of 2 dpt, where the addition value always indicates the difference between the utilitarian values of the refractive power at the far reference point  $B_F$  and at the near reference point  $B_N$ .

For all figures the choice of coordinates is such that the zero point  $(x, y) = (0, 0)$  is located 4 mm below the centering point  $B_Z$ , where  $x$  is the horizontal direction and  $y$  is the vertical direction in the use position, as is usually the case. In addition, the surface astigmatism as referred to below is to be understood as being a vectorial surface astigmatism as defined in equation (3).

The illustrations according to the state of the art are always labeled as FIG A, while those according to exemplary embodiment 1 are labeled as FIG B and those according to exemplary embodiment 2 are labeled as FIG C. The corresponding depths of camber in mm as a function of the coordinates  $(x, y)$  in mm of exemplary embodiment 1 and exemplary embodiment 2 according to this invention are also disclosed in Tables 3A and 3B. Table 3A gives the depth of camber for the progressive surface in mm according to exemplary embodiment 1 and Table 3B gives the depth of camber for the progressive lens according to exemplary embodiment 2 of the present invention.



In this situation, a lens is completely characterized only when the back surface properties and other parameters (given in Table 1) have also been specified in addition to the front surface properties. Table 1 shows a complete characterization of the inventive lenses of exemplary embodiments 1 and 2 as well as those according to the state of the art. In the present case, the formula gives a cylinder value of zero, so the back surfaces of the lenses have a cylinder value of zero. Therefore only their surface refractive value is given. The front surfaces are adjusted according to basic curves. The fact that the front surface of the lens according to the state of the art has a greater basic curve than do the lenses according to this invention (5.7 dpt instead of 4.0 dpt) is due to the shallower deflection of the lenses according to this invention. However, this difference has no effect on the distribution of the front surface of astigmatism and thus on the comparability of this invention with the state of the art. It is essential for comparability that the invention and the state of the art belong to the same prescription, i.e., have the same prescription values.

Table 1.

Property	State of the art (Progressive Life 1)	Invention (Ex 1)	Invention (Ex 2)
<b>Surfaces</b>			
Basic curve	5.7	4.0	4.0
Surface refractive value Front surface (dpt) in $B_F$	6.5	4.5	4.5
Surface refractive value Back surface (dpt)	-5.82	-4.05	-4.10
<b>Prescription data</b>			
Sphere (dpt)	0.5	0.5	0.5
Cylinder (dpt)	0.0	0.0	0.0
Axis (deg)	0.0	0.0	0.0
Addition (dpt)	2.0	2.0	2.0
Prism (cm/m)	0.0	0.0	0.0
<b>Centering</b>			
Interpupillary distance (mm)	63.0	64.0	64.0
Corneal vertex distance (mm)	15.0	13.0	13.0
Anterior tilt (deg)	8.0	7.0	7.0
Ocular pivot point distance (mm)	28.5	26.5	26.5
<b>Stamp points</b>			
Far reference point (x, y)	(0.0, 8.0)	(0.0, 8.0)	(0.0, 8.0)
Centering point (x, y)	(0.0, 4.0)	(0.0, 4.0)	(0.0, 4.0)
Near reference point (x, y)	(-2.5, -14.0)	(-2.5, -14.0)	(-2.5, -14.0)
Prism reference point (x, y)	(0.0, 0.0)	(0.0, 0.0)	(0.0, 0.0)
<b>Material</b>			
Name	Perfalit 1.5	Perfalit 1.6	Perfalit
Refractive index n	1.502	1.597	1.502
<b>Lens data</b>			
Center thickness d (mm)	4.7	2.34	2.6
Diameter (mm)	80.0	65.0	65.0
Thickness reduction prism (cm/m)	0.0	1.0	0.0

The lenses are first characterized by certain refractive value distributions which are shown in FIG 1A for the state of the art, in FIG 1B for exemplary embodiment 1 and in FIG 1C for exemplary embodiment 2. FIGS 1A through 1C show the distribution of the mean surface refractive value  $D = (n - 1)(k_1 + k_2)/2$  of the front surface as a function of the coordinates (x, y) in mm, where n denotes the refractive index and  $k_{1,2}$  denote the main curvatures of the front surface. The distribution of the mean refractive value is depicted as isolines and/or height lines of the same value at a distance of 0.25 dpt.

As shown by FIGS 1A through 1C, the increase in the surface refractive value from 4.5 dpt in the far vision part to 6.25 dpt in the near

vision part in the lenses according to the present invention is less than in the case of the lens according to the state of the art which shows an increase from 5.25 dpt to 7.25 dpt, so the surface astigmatism of the front surface is also lower than that in the state of the art. One advantage of this invention is that this, together with the spherical back surface, achieves the same addition in the use position of 2.0 dpt as in the state of the art but without the astigmatism in the use position, achieving the high values as in the state of the art due to the addition and the back surface. In particular, because of the inclined ray passage, the addition of 2.0 dpt in the use position is achieved even if the increase in surface refractive value of the front surface is less than 2.0 dpt. The maximum use value of the astigmatism of the lenses according to the invention is 2.41 dpt for exemplary embodiment 1 and 2.53 dpt for exemplary embodiment 2 in comparison with 2.92 dpt in the state of the art.

FIGS 2A through 2C show the plot of the absolute value of the astigmatism of the front surface according to equation (3) as a function of the coordinates (x, y) in mm, where FIG 2A shows the state of the art, FIG 2B shows exemplary embodiment 1 and FIG 2C shows exemplary embodiment 2. The course of the absolute value of the surface astigmatism is represented by isolines at a distance of 0.25 dpt. All the information that can be obtained by considering the absolute value of the astigmatism  $|\bar{A}|$  can be seen from FIGS 2A through 2C.

As FIG 2A shows, in the state of the art the surface astigmatism assumes an absolute value of max 2.62 dpt according to equation (3). In comparison with that, the maximum values of the surface astigmatism amount to 2.04 dpt for exemplary embodiment 1 (FIG 2B) and 2.16 dpt for exemplary embodiment 2 (FIG 2C) of this invention.

FIGS 3A through 3C show the distribution of the gradient  $|\text{grad}(|\bar{A}|)|$  in dpt/mm of the absolute value of the astigmatism of the front surface as a function of the coordinates (x, y) in mm for the state of the art (FIG 3A) for exemplary embodiment 1 (FIG 3B) and for exemplary embodiment 2 (FIG 3C). The distribution of the gradient is represented by isolines at a distance of 0.05 dpt/mm.

In comparison with the distribution of the absolute value of the astigmatism which is shown in FIGS 2A through 2C, a diagram of the distribution of the total vector field  $\vec{A}$  of the astigmatism also contains the direction information. FIGS 4A through 4C show the vector field of the surface astigmatism of the front surface as a function of the coordinates  $(x, y)$  in mm for the state of the art (FIG 4A), for exemplary embodiment 1 (FIG 4B) and exemplary embodiment 2 (FIG 4C). Accordingly, FIG 5A through 5C show the distribution of the rotation of the astigmatism  $\text{rot}\vec{A}$  in dpt/mm as a function of the coordinates  $(x, y)$  in mm for the state of the art (FIG 5A), for exemplary embodiment 1 (FIG 5B) and for exemplary embodiment 2 (FIG 5C) represented by isolines. FIGS 6A through 6C show the distribution of the divergence  $\text{div}\vec{A}$  in dpt/mm as a function of the coordinates  $(x, y)$  in mm for the state of the art (FIG 6A), for exemplary embodiment 1 (FIG 6B) and exemplary embodiment 2 (FIG 6C) in the form of isolines.

It can be seen from FIG 4A for the state of the art that the vector cambers of the astigmatism  $\vec{A}$  in the region around  $(x, y) = (20, -6)$  mm all have approximately the same length but vary greatly in direction. However, this variation cannot be perceived on the basis of the diagram of the absolute value of the astigmatism  $|\vec{A}|$ . For example, the size of the gradient of the absolute value  $|\text{grad}(|\vec{A}|)|$  in the region around  $(x, y) = (20, -6)$ , as shown in FIG 3A assumes values which are not above 0.1 dpt/mm. The great variations in the astigmatism  $\vec{A}$  become visible only through the size of the rotation of the astigmatism  $\text{rot}\vec{A}$  which assumes a value of more than 0.4 dpt/mm, as shown in FIG 5A for example. In comparison with that, the size of rotation of the astigmatism  $\text{rot}\vec{A}$  in the same region  $(x, y) = (20, -6)$  for exemplary embodiment 1 (FIG 5B) and for exemplary embodiment 2 (FIG 5C) of the invention are of an order of magnitude of only 0.15 dpt/mm.

Other quantitative features of the divergence  $\text{div}\vec{A}$  and rotation  $\text{rot}\vec{A}$  of the astigmatism of the lenses according to exemplary embodiment 1 and 2 as well as the state of the art are illustrated below on the basis of horizontal sections ( $x$ : variable,  $y = \text{const}$ ).

In each case, FIG -A (FIGS 7A through 11A) is based on the state of the art, FIG -B (FIGS 7B through 11B) is based on the first exemplary embodiment and FIG -C (FIGS 7C through 11C) is based on the second exemplary embodiment.

FIG 7D is based on a series with additions of 2.0, 2.5 and 3.0 dpt of front surfaces of the same type of design as the front surface in exemplary embodiment 2.

In each case, FIG -D (FIGS 8D through 11D) is based on a series of front surfaces of the same design type as the front surface of exemplary embodiment 1, said front surfaces differing only through their additions in the use position. The series of front surfaces shown here has additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt accordingly.

In each case, FIG -E (FIGS 8E through 11E) is based on a series of front surfaces of the same type of design as the front surface of exemplary embodiment 2, said front surfaces differing only in their additions in the use position. The series of front surfaces shown here has additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt accordingly.

#### **Positions of the extremes of $\text{div}\bar{A}$**

The rotation  $\text{rot}\bar{A}$  and divergence  $\text{div}\bar{A}$  of the astigmatism should be minimized in a good lens. Since  $\text{div}\bar{A}$  normally assumes extreme values within a lens, a design is better when these extremes are located farther toward the outer periphery of the lens.

In the state of the art, the divergence of the astigmatism  $\text{div}\bar{A}$  has many pronounced extremes that are close together as shown in FIG 6A. In comparison with that, the distribution of the divergence of the astigmatism  $\text{div}\bar{A}$  for front surfaces according to this invention is very smooth (see FIGS 6B and 6C for exemplary embodiment 1 and/or exemplary embodiment 2). There are either no extremes close to the center of the lens or they are hardly pronounced. The only mentionable extremes are definitely outside of the range  $y \geq -8$  mm. The term "mentionable" refers to an extreme the absolute value of which is more than (0.1/mm) times the addition. In exemplary embodiment 1 the maximum value of the

divergence of the astigmatism  $|\text{div}\bar{A}|$  is assumed at the minimum at  $(x, y) = (11.3, -11.9)$  mm and amounts to 0.31 dpt/mm. In exemplary embodiment 2 the maximum value of 0.39 dpt/mm is reached at  $(x, y) = (11.9, -13.8)$ . In comparison with that the maximum value of the divergence  $|\text{div}\bar{A}|$  for the state of the art is 0.6 dpt/mm and is assumed to be much closer to the midpoint of the lens at the point  $(x, y) = (6.1, -8.3)$ .

The property of no (or no mentionable) extremes of the divergence of the astigmatism  $\text{div}\bar{A}$  being inside the range  $y \geq -8$  mm is applicable to all additions of the design according to this invention in the series 1.0, 2.0, 2.5, 3.0, 3.5 dpt (see Table 2A and Table 2B). Table 2A shows the positions of the extremes (maximum and minimum) of the divergence of the astigmatism  $\text{div}\bar{A}$  for a lens according to exemplary embodiment 1 of the invention and Table 2B shows the positions of the extremes of the divergence of the astigmatism  $\text{div}\bar{A}$  for a lens according to exemplary embodiment 2 of this invention. Even the range  $y \geq -9$  mm is free of extremes in the series beyond the addition 2.0.

Table 2A:

Addition	Minimum (dpt/mm)	Maximum (dpt/mm)
	Position (x, y) (mm)	Position (x, y) (mm)
1.0	$\frac{-0.181}{(6.0, -9.4)}$	$\frac{0.171}{(-1.5, -9.1)}$
2.0	$\frac{-0.309}{(11.3, -11.9)}$	$\frac{0.287}{(-7.0, -13.4)}$
2.5	$\frac{-0.373}{(11.5, -12.1)}$	$\frac{0.344}{(-7.0, -13.6)}$
3.0	$\frac{-0.440}{(12.0, -11.8)}$	$\frac{0.406}{(-7.0, -13.9)}$
3.5	$\frac{-0.502}{(12.8, -12.0)}$	$\frac{0.452}{(-6.3, -14.0)}$

Table 2B:

Addition	Minimum (dpt/mm)	Maximum (dpt/mm)
	Position (x, y) (mm)	Position (x, y) (mm)
1.0	$\frac{-0.191}{(5.0, -8.4)}$	$\frac{0.164}{(-3.0, -13.0)}$
2.0	$\frac{-0.386}{(11.9, -13.8)}$	$\frac{0.372}{(-6.7, -14.5)}$
2.5	$\frac{-0.508}{(10.9, -13.4)}$	$\frac{0.486}{(-6.5, -15.2)}$
3.0	$\frac{-0.668}{(11.0, -13.8)}$	$\frac{0.590}{(-5.4, -15.6)}$
3.5	$\frac{-0.808}{(10.2, -12.0)}$	$\frac{0.712}{(-3.8, -15.5)}$

**Monotonic change in rotation of the astigmatism  $|\text{rot}\bar{A}|$  in the horizontal section**

Variations in the rotation of the astigmatism  $|\text{rot}\bar{A}|$  close to the center of the lens are especially problematical. Therefore, a lens with a negligible value of  $|\text{rot}\bar{A}|$  at  $x = 0$  mm and the gradual and most uniform possible increase in the direction of finite  $x$  values is desirable.

In contrast with the state of art in particular, in the far vision part of the lens according to this invention, there is a horizontal section along which the increase in  $|\text{rot}\bar{A}|$  is monotonic over a range covering at least  $|x| \leq 16$  mm. The exact position of the horizontal section depends on the design and varies between  $y = 3$  mm and  $y = 5$  mm. Since the lenses according to this invention (exemplary embodiment 1 and exemplary embodiment 2) and according to the state of the art have different designs, the horizontal section for exemplary embodiment 1 is at  $y = 4.8$  mm (FIG 7B), that for exemplary embodiment 2 is at  $y = 3$  mm (FIG 7C) and the corresponding horizontal section for the state of the art is at  $y = 3$  mm (FIG 7A).

FIG 7B shows a horizontal section of the absolute value of the rotation of the astigmatism  $|\text{rot}\bar{A}|$  for exemplary embodiment 1 of the invention  $y = 4.8$  mm in comparison with the corresponding section in the state of the art (FIG 7A). For an inventive lens according to exemplary embodiment 1, the value of  $|\text{rot}\bar{A}|$  at  $x = 0$  mm has dropped to 30% of the previous value (FIG 7B) and the increase for finite  $x$  values is gradual

and monotonic in the entire range  $|x| \leq 16$  mm, whereas in the state of the art, there are great fluctuations and multiple extremes in the same range.

For the lens according to exemplary embodiment 2 of this invention, the monotonic area is even located at  $y = 3$  mm, i.e., closer to the center of the lens, and the monotonic property even extends to the area  $|x| \leq 20$  mm as depicted in FIG 7C.

Even if the absolute value of the rotation of the astigmatism  $|\text{rot}\bar{A}|$  depends on the addition, the feature of the monotonic increase is preserved, at least for exemplary embodiment 2 in the range  $|x| \leq 20$  mm for multiple additions almost unchanged. Besides addition 2.0, the design according to this invention for exemplary embodiment 2 also meets the monotonic requirement for addition 2.5 dpt within the series with additions of 1.0, 2.0, 2.5, 3.0, 3.5 dpt and for the addition 3.0 the dependence  $|\text{rot}\bar{A}|$  does not have any mentionable extremes in the range  $|x| \leq 20$  mm as shown in FIG 7D.

#### **Extreme values of rotation and divergence of astigmatism**

This invention is compared below with the state of the art with respect to maximum values assumed by the absolute value of rotation  $|\text{rot}\bar{A}|$  and divergence  $\text{div}\bar{A}$  of the astigmatism along characteristic horizontal sections ( $x, y = \text{const}$ ).

FIGS 8A through 8D show the plot of the absolute value of the rotation of the astigmatism in dpt/mm in the horizontal section at  $y = +6$  mm. As shown in FIG 8A, the absolute value of the rotation of the astigmatism  $|\text{rot}\bar{A}|$  in the horizontal section at  $y = +6$  mm varies greatly for the state of the art, assuming values of more than 0.3 dpt/mm at the maximum. By comparison, the absolute value of the rotation of the astigmatism  $|\text{rot}\bar{A}|$  amounts to less than 0.03 dpt/mm close to the principal line in the case of this invention for exemplary embodiment 1 and does not increase beyond 0.12 dpt/mm along the entire horizontal section, as shown in FIG 8B. For exemplary embodiment 2,  $|\text{rot}\bar{A}|$  also falls below a value of 0.03 dpt/mm near the principal line and does not



increase above 0.12 dpt/mm along the entire horizontal section, as shown in FIG 8C.

The quantity  $|\text{rot}\bar{A}|$  is scaled approximately linearly with the addition, as shown FIG 8D (exemplary embodiment 1) and FIG 8E (exemplary embodiment 2) for a series of lenses with additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt. In particular the maximum value is scaled linearly, namely according to the following dependence for exemplary embodiment 1:

$$|\text{rot}\bar{A}|_{\max} \approx 0.06 \text{ addition/dpt}\cdot\text{dpt/mm}$$

and according to the following dependence for exemplary embodiment 2:

$$|\text{rot}\bar{A}|_{\max} \approx 0.07 \text{ addition/dpt}\cdot\text{dpt/mm}.$$

FIGS 9A through 9E show the plot of the absolute value of the rotation of the astigmatism  $|\text{rot}\bar{A}|$  in dpt/mm in the section at  $y = -14$  mm. FIGS 9A through 9C show the plot of  $|\text{rot}\bar{A}|$  for a lens with addition of 2.0 dpt according to the state of the art (FIG 9A), according to exemplary embodiment 1 (FIG 9B) and according to exemplary embodiment 2 (FIG 9C). FIG 9D (exemplary embodiment 1) and FIG 9E (exemplary embodiment 2) show the plot of  $|\text{rot}\bar{A}|$  for a series of lenses with additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt.

The plot of  $|\text{rot}\bar{A}|$  in FIG 9A is characterized by clearly pronounced maximums with values of more than 0.5 dpt/mm and steep flanks, but the plot of FIG 9B is characterized by weakly pronounced maximums with values of less than 0.17 dpt/mm. The same thing is true of the plot in FIG 9C where the maximums have values of less than 0.16 dpt/mm. Furthermore, the plot is almost like a plateau at  $-x$  values.

As FIGS 9A through 9C show,  $\text{rot}\bar{A}$  fluctuates much more in the state of the art than in the case of the present invention. The quantity  $\text{rot}\bar{A}$  is also scaled approximately linearly with the addition at  $y = -14$  mm, as shown in FIG 9D and FIG 9E for a series of lenses according to exemplary embodiment 1 and exemplary embodiment 2, respectively, with different additions. The value of the maximum in particular is scaled

linearly, namely for exemplary embodiment 1 according to the dependence:

$$|\text{rot}\bar{A}|_{\max} \approx 0.08 \text{ addition/dpt}\cdot\text{dpt}/\text{mm}$$

and for exemplary embodiment 2 according to the dependence:

$$|\text{rot}\bar{A}|_{\max} \approx 0.07 \text{ addition/dpt}\cdot\text{dpt}/\text{mm}.$$

FIGS 10A through 10E show the plot of the divergence of the astigmatism  $\text{div}\bar{A}$  in dpt/mm in the horizontal section at  $y = 0$  mm. As shown in FIG 10A, the plot is characterized by clearly pronounced maximums and minimums of  $\text{div}\bar{A}$  with steep flanks and with values of more than 0.25 dpt/mm and less than -0.25 dpt/mm. However, the maximum of  $\text{div}\bar{A}$  has a value of less than 0.18 dpt/mm and the minimum has a value greater than -0.18 dpt/mm for exemplary embodiment 1. Exemplary embodiment 2 even has smaller values for the maximum and minimum. The maximum of  $\text{div}\bar{A}$  in exemplary embodiment 2 is less than 0.11 dpt/mm and the minimum is greater than -0.11 dpt/mm.

As shown in FIGS 10A through 10C, the improvement from the state of the art to the present invention is especially noteworthy with regard to the quantity  $\text{div}\bar{A}$  at  $y = 0$  mm. An uneven behavior with multiple extremes and steep flanks and absolute values of more than 0.25 dpt/mm as is characteristic of the state of the art (FIG 10A) is replaced in exemplary embodiment 1, as shown in FIG 10B by one maximum and one minimum with values of less 0.17 dpt/mm for  $|x| \leq 5$  mm and a plateau-shaped tail with values of less than 0.06 dpt/mm. For exemplary embodiment 2, the absolute values of the extreme are even below 0.11 dpt/mm and the flanks do not exceed the absolute value of 0.011 dpt/mm (FIG 10C).

The quantity  $\text{div}\bar{A}$  is not scaled linearly at  $y = 0$  mm but is consistent with the addition, as illustrated in FIG 10D (exemplary embodiment 1) and FIG 10E (exemplary embodiment 2) for inventive lenses with addition of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt. The value of the maximum is scaled according to the following dependence for exemplary embodiment 1:

$$(\text{div}\bar{A})_{\max} \approx (0.08 \text{ addition/dpt} + 0.03) \text{ dpt/mm}$$

and the value of the minimum is scaled according to:

$$(\text{div}\bar{A})_{\min} \approx (-0.05 \text{ addition/dpt} - 0.08) \text{ dpt/mm}.$$

For exemplary embodiment 2 it holds that:

$$(\text{div}\bar{A})_{\max} \approx (0.042 \text{ addition/dpt} + 0.029) \text{ dpt/mm}$$

$$(\text{div}\bar{A})_{\min} \approx (-0.022 \text{ addition/dpt} - 0.06) \text{ dpt/mm}.$$

FIGS 11A through 11E show the plot of the divergence of the astigmatism  $\text{div}\bar{A}$  in dpt/mm in the horizontal section at  $y = -14$  mm. As shown in FIG 11A, the plot of the divergence at  $y = -14$  mm is characterized by clearly pronounced maximums and minimums with steep flanks and values of more than 0.45 dpt/mm and less than -0.6 dpt/mm. However, the plot of the divergence of the inventive exemplary embodiments is much smoother, whereby for exemplary embodiment 1 the maximum is below a value of 0.3 dpt/mm and the minimum is above a value of -0.35 dpt/mm. For exemplary embodiment 2 the maximum has a value of 0.4 dpt/mm the minimum has a value of more than -0.4 dpt/mm.

FIG 11D (exemplary embodiment 1) and FIG 11E (exemplary embodiment 2) show the plot of the divergence of the astigmatism for a series of lenses with additions of 1.0, 2.0, 2.5, 3.0 and 3.5 dpt. As in a horizontal section at  $y = 0$  mm, the quantity  $\text{div}\bar{A}$  is also scaled similarly at  $y = -14$  mm for exemplary embodiment 1 with the addition as shown in FIG 11D for the additions 1.0, 2.0, 2.5, 3.0 and 3.5 dpt. For exemplary embodiment 2 the relationship is linear (see FIG 11E) but leads to higher absolute values of  $\text{div}\bar{A}$  than in the case of exemplary embodiment 1. For the maximum of  $\text{div}\bar{A}$  in exemplary embodiment 1, the following holds:

$$(\text{div}\bar{A})_{\max} \approx (0.12 \text{ addition/dpt} + 0.06) \text{ dpt/mm},$$

but for the minimum:

$$(\text{div}\bar{A})_{\min} \approx (-0.13 \text{ addition/dpt} - 0.05) \text{ dpt/mm}.$$

For exemplary embodiment 2 it holds that:

$$(\operatorname{div} \bar{A})_{\max} \approx 0.2 \text{ addition/dpt}\cdot\text{dpt/mm}$$

$$(\operatorname{div} \bar{A})_{\min} \approx -0.22 \text{ addition/dpt}\cdot\text{dpt/mm}.$$

Table 3A

X/Y	-30,0	-27,5	-25,0	-22,5	-20,0	-17,5	-15,0	-12,5	-10,0	-7,5	-5,0	-2,5	0,0
30,0	7,01201	6,43297	5,90677	5,43260	5,00970	4,63738	4,31515	4,04262	3,81937	3,64506	3,51945	3,44238	3,41370
27,5	6,45126	5,87398	5,34940	4,87673	4,45520	4,08414	3,76295	3,49122	3,26854	3,09458	2,96913	2,89199	2,86301
25,0	5,94229	5,36642	4,84311	4,37160	3,95115	3,58108	3,26080	2,98982	2,76771	2,59413	2,46884	2,39164	2,36240
22,5	5,48467	4,90984	4,38746	3,91676	3,49705	3,12769	2,80810	2,53778	2,31621	2,14301	2,01789	1,94067	1,91121
20,0	5,07803	4,50393	3,98210	3,51184	3,09249	2,72352	2,40439	2,13453	1,91342	1,74059	1,61569	1,53850	1,50886
17,5	4,72215	4,14848	3,62683	3,15656	2,73718	2,36829	2,04933	1,77972	1,55890	1,38637	1,26170	1,18459	1,15483
15,0	4,41704	3,84349	3,32158	2,85088	2,43108	2,06188	1,74274	1,47309	1,25235	1,07998	0,95551	0,87851	0,84867
12,5	4,16294	3,58917	3,06654	2,59498	2,17438	1,80444	1,48467	1,21458	0,99358	0,82115	0,69675	0,61985	0,58998
10,0	3,96016	3,38571	2,86204	2,38928	1,96748	1,59637	1,27550	1,00441	0,78266	0,60977	0,48519	0,40831	0,37843
7,5	3,80967	3,23383	2,70868	2,23440	1,81101	1,43827	1,11581	0,84320	0,62004	0,44599	0,32070	0,24362	0,21371
5,0	3,71317	3,13537	2,60807	2,13164	1,70809	1,33117	1,00651	0,73172	0,50645	0,33048	0,20363	0,12573	0,09567
2,5	3,67289	3,09268	2,56284	2,08367	1,65527	1,27741	0,94974	0,67185	0,44341	0,26429	0,13470	0,05500	0,02441
0,0	3,69136	3,10822	2,57551	2,09317	1,66132	1,27983	0,94829	0,66625	0,43335	0,24959	0,11561	0,03249	0,00048
-2,5	3,77113	3,18444	2,64841	2,16252	1,72669	1,34085	1,00453	0,71718	0,47843	0,28849	0,14858	0,06048	0,02564
-5,0	3,91476	3,32393	2,78382	2,29382	1,85346	1,46248	1,12048	0,82676	0,58095	0,38359	0,23657	0,14245	0,10365
-7,5	4,12320	3,52790	2,98338	2,48872	2,04319	1,64635	1,29779	0,99689	0,74323	0,53769	0,38294	0,28243	0,23934
-10,0	4,39668	3,79639	3,24684	2,74690	2,29575	1,89268	1,53713	1,22838	0,96640	0,75255	0,59019	0,48365	0,43671
-12,5	4,73486	4,12888	3,57342	3,06742	2,60998	2,20026	1,83740	1,52047	1,25014	1,02850	0,85958	0,74823	0,69854
-15,0	5,13735	4,52477	3,96243	3,44945	2,98485	2,56766	2,19689	1,87204	1,59445	1,36663	1,19286	1,07812	1,02670
-17,5	5,60350	4,98305	4,41272	3,89182	3,41930	2,99417	2,61592	2,28437	2,00117	1,76898	1,59198	1,47510	1,42263
-20,0	6,13248	5,50293	4,92400	4,39476	3,91409	3,48151	3,09679	2,75975	2,47238	2,23735	2,05851	1,94042	1,88720
-22,5	6,72414	6,08520	5,49741	4,95974	4,47146	4,03220	3,64182	3,30038	3,00994	2,77320	2,59354	2,47498	2,42147
-25,0	7,38002	6,73144	6,13480	5,58921	5,09398	4,64877	4,25345	3,90837	3,61559	3,37784	3,19806	3,07971	3,02648
-27,5	8,10252	7,44403	6,83876	6,28578	5,78429	5,33370	4,93397	4,58577	4,29112	4,05266	3,87311	3,75547	3,70299
-30,0	8,89531	8,22653	7,61273	7,05260	6,54507	6,08942	5,68566	5,33480	5,03857	4,79945	4,62007	4,50324	4,45190
X/Y	2,5	5,0	7,5	10,0	12,5	15,0	17,5	20,0	22,5	25,0	27,5	30,0	
30,0	3,43327	3,50112	3,61754	3,78293	3,99778	4,26263	4,57809	4,94480	5,36347	5,83487	6,35980	6,93907	
27,5	2,88204	2,94911	3,06456	3,22882	3,44240	3,70590	4,01992	4,38516	4,80234	5,27222	5,79559	6,37326	
25,0	2,38093	2,44730	2,56189	2,72513	2,93759	3,19989	3,51269	3,87670	4,29266	4,76133	5,28348	5,85992	
22,5	1,92930	1,99506	2,10889	2,27127	2,48279	2,74411	3,05592	3,41899	3,83405	4,30185	4,82314	5,39869	
20,0	1,52657	1,59181	1,70500	1,86667	2,07746	2,33806	2,64920	3,01168	3,42622	3,89354	4,41436	4,98938	
17,5	1,17223	1,23702	1,34969	1,51085	1,72117	1,98136	2,29221	2,65451	3,06899	3,53631	4,05706	4,63194	
15,0	0,86582	0,93024	1,04255	1,20343	1,41359	1,67379	1,98481	2,34742	2,76234	3,23019	3,75137	4,32646	
12,5	0,60693	0,67110	0,78325	0,94416	1,15462	1,41536	1,72712	2,09060	2,50646	2,97531	3,49742	4,07311	
10,0	0,39525	0,45929	0,57161	0,73307	0,94446	1,20641	1,51953	1,88444	2,30178	2,77209	3,29556	3,87253	
7,5	0,23049	0,29470	0,40778	0,57059	0,78370	1,04756	1,36270	1,72967	2,14906	2,62137	3,14701	3,72608	
5,0	0,11254	0,17752	0,29230	0,45743	0,67321	0,93990	1,25788	1,62768	2,04987	2,52502	3,05363	3,63552	
2,5	0,04163	0,10826	0,22601	0,39496	0,61483	0,88556	1,20746	1,58106	2,00695	2,48576	3,01789	3,60302	
0,0	0,01837	0,08788	0,21031	0,38495	0,61068	0,88695	1,21398	1,59236	2,02279	2,50596	3,04211	3,63084	
-2,5	0,04431	0,11771	0,24619	0,42838	0,66194	0,94550	1,27910	1,66340	2,09927	2,58747	3,12811	3,72121	
-5,0	0,12251	0,20000	0,33515	0,52591	0,76895	1,06183	1,40343	1,79501	2,23746	2,73156	3,27749	3,87579	
-7,5	0,25746	0,33838	0,47979	0,67894	0,93194	1,23494	1,58645	1,98688	2,43737	2,93857	3,49084	4,09522	
-10,0	0,45377	0,53676	0,68318	0,88953	1,15173	1,46512	1,82711	2,23739	2,69683	3,20629	3,76642	4,37838	

-12,5	0,71470	0,79855	0,94818	1,15987	1,42954	1,75226	2,12447	2,54487	3,01371	3,53215	4,10126	4,72211
-15,0	1,04225	1,12639	1,27772	1,49296	1,76798	2,09792	2,47879	2,90874	3,38722	3,91521	4,49402	5,12452
-17,5	1,43771	1,52195	1,67420	1,89161	2,17018	2,50527	2,89290	3,33095	3,81837	4,35581	4,94447	5,58532
-20,0	1,90190	1,98622	2,13909	2,35779	2,63857	2,97718	3,36998	3,81475	4,31014	4,85633	5,45418	6,10517
-22,5	2,43600	2,52033	2,67349	2,89288	3,17496	3,51598	3,91280	4,36323	4,86571	5,41989	6,02648	6,68710
-25,0	3,04117	3,12543	3,27850	3,49807	3,78087	4,12366	4,52375	4,97906	5,48813	6,05011	6,68536	7,33549
-27,5	3,71829	3,80244	3,95512	4,17451	4,45784	4,80218	5,20515	5,66492	6,18020	6,75011	7,37454	8,05484
-30,0	4,46795	4,55223	4,70446	4,92368	5,20766	5,55367	5,95950	6,42364	6,94508	7,52312	8,15756	8,84917

Table 3B

X/Y	-30,0	-27,5	-25,0	-22,5	-20,0	-17,5	-15,0	-12,5	-10,0	-7,5	-5,0	-2,5	0,0
30,0	7,07685	6,50602	5,98575	5,51551	5,09488	4,72350	4,40110	4,12747	3,90253	3,72631	3,59880	3,51939	3,48743
27,5	6,50679	5,93804	5,41954	4,95076	4,53130	4,16085	3,83915	3,56607	3,34150	3,16540	3,03781	2,95833	2,92640
25,0	5,98922	5,42224	4,90519	4,43761	4,01910	3,64936	3,32820	3,05553	2,83124	2,65525	2,52756	2,44798	2,41607
22,5	5,52352	4,95799	4,44212	3,97547	3,55767	3,18845	2,86768	2,59527	2,37117	2,19527	2,06750	1,98777	1,95582
20,0	5,10919	4,54479	4,02982	3,56384	3,14652	2,77762	2,45705	2,18478	1,96074	1,78486	1,65703	1,57716	1,54510
17,5	4,74581	4,18223	3,66786	3,20229	2,78520	2,41641	2,09587	1,82357	1,59950	1,42357	1,29566	1,21567	1,18351
15,0	4,43296	3,86999	3,35594	2,89049	2,47338	2,10446	1,78373	1,51124	1,28701	1,11097	0,98298	0,90291	0,87065
12,5	4,17021	3,60775	3,09386	2,62830	2,21089	1,84160	1,52046	1,24755	1,02296	0,84667	0,71856	0,63847	0,60615
10,0	3,95747	3,39533	2,88152	2,41570	1,99783	1,62794	1,30611	1,03251	0,80728	0,63052	0,50214	0,42196	0,38965
7,5	3,79556	3,23331	2,71924	2,25304	1,83454	1,46384	1,14105	0,86640	0,64018	0,46259	0,33367	0,25326	0,22092
5,0	3,68570	3,12283	2,60805	2,14099	1,72168	1,34999	1,02603	0,75002	0,52235	0,34339	0,21339	0,13240	0,09991
2,5	3,62990	3,06579	2,54974	2,08141	1,66084	1,28786	0,96243	0,68470	0,45503	0,27393	0,14194	0,05951	0,02653
0,0	3,63076	3,06489	2,54712	2,07716	1,65493	1,28030	0,95309	0,67320	0,44076	0,25628	0,12074	0,03543	0,00105
-2,5	3,69140	3,12333	2,60340	2,13144	1,70726	1,33062	1,00116	0,71840	0,48206	0,29265	0,15183	0,06189	0,02469
-5,0	3,81473	3,24419	2,72185	2,24760	1,82115	1,44193	1,10928	0,82249	0,58103	0,38539	0,23799	0,14221	0,10103
-7,5	4,00175	3,42901	2,90460	2,42813	1,99898	1,61636	1,27944	0,98743	0,73985	0,53723	0,38263	0,28055	0,23509
-10,0	4,25267	3,67813	3,15172	2,67277	2,24042	1,85375	1,51200	1,21440	0,96026	0,75025	0,58841	0,48032	0,43108
-12,5	4,56762	3,99126	3,46248	2,98040	2,54420	2,15303	1,80624	1,50298	1,24216	1,02488	0,85655	0,74372	0,69181
-15,0	4,94702	4,36806	3,83608	3,35018	2,90953	2,51353	2,16138	1,85211	1,58483	1,36165	1,18858	1,07254	1,01907
-17,5	5,39017	4,80762	4,27161	3,78129	3,33580	2,93453	2,57676	2,26179	1,98939	1,76218	1,58619	1,46832	1,41413
-20,0	5,89620	5,30922	4,76859	4,27332	3,82264	3,41605	3,05303	2,73342	2,45745	2,22794	2,05063	1,93203	1,87760
-22,5	6,46489	5,87267	5,32677	4,82629	4,37055	3,95901	3,59165	3,26856	2,99043	2,76002	2,58265	2,46421	2,40997
-25,0	7,09666	6,49805	5,94644	5,44081	4,98018	4,56451	4,19408	3,88893	3,58978	3,35937	3,18269	3,06514	3,01162
-27,5	7,79271	7,18673	6,62886	6,11786	5,65292	5,23424	4,86207	4,53601	4,25651	4,02667	3,85116	3,73490	3,68238
-30,0	8,55550	7,94149	7,37659	6,86008	6,39148	5,97079	5,59766	5,27113	4,99167	4,76277	4,58872	4,47398	4,42258

X/Y	2,5	5,0	7,5	10,0	12,5	15,0	17,5	20,0	22,5	25,0	27,5	30,0
30,0	3,50292	3,56618	3,67806	3,83929	4,04997	4,31020	4,62019	4,98011	5,39018	5,85068	6,36195	6,92447
27,5	2,94190	3,00511	3,11698	3,27808	3,48841	3,74809	4,05734	4,41634	4,82533	5,28454	5,79435	6,35524
25,0	2,43150	2,49474	2,60665	2,76762	2,97763	3,23687	3,54555	3,90389	4,31205	4,77031	5,27903	5,83867
22,5	1,97117	2,03453	2,14652	2,30730	2,51705	2,77598	3,08431	3,44223	3,84989	4,30755	4,81556	5,37433
20,0	1,56049	1,62398	1,73593	1,89656	2,10614	2,36496	2,67319	3,03099	3,43848	3,89591	4,40356	4,96177
17,5	1,19899	1,26251	1,37437	1,53492	1,74453	2,00348	2,31190	2,66990	3,07757	3,53508	4,04288	4,60064
15,0	0,88614	0,94962	1,06145	1,22211	1,43199	1,69134	2,00025	2,35877	2,76692	3,22483	3,73267	4,29065
12,5	0,62156	0,68499	0,79695	0,95793	1,16837	1,42845	1,73817	2,09752	2,50649	2,96507	3,47336	4,03150
10,0	0,40500	0,46847	0,58074	0,74237	0,95373	1,21492	1,52583	1,88634	2,29636	2,75583	3,26474	3,82330
7,5	0,23630	0,29997	0,41289	0,57561	0,78841	1,05119	1,38368	1,72567	2,13702	2,59758	3,10754	3,66708
5,0	0,11538	0,17964	0,29380	0,45830	0,67318	0,93807	1,25255	1,61634	2,02930	2,49152	3,00309	3,56422
2,5	0,04227	0,10781	0,22421	0,39155	0,60939	0,87708	1,19408	1,56013	1,97524	2,43963	2,95341	3,51677
0,0	0,01724	0,08515	0,20546	0,37738	0,59960	0,87103	1,19117	1,55999	1,97776	2,44477	2,96122	3,52726
-2,5	0,04132	0,11282	0,23878	0,41737	0,64590	0,92251	1,24673	1,61898	2,03988	2,50993	3,02948	3,59857
-5,0	0,11759	0,19305	0,32558	0,51234	0,74913	1,03280	1,38270	1,73958	2,16439	2,63801	3,16105	3,73348
-7,5	0,25069	0,32948	0,46824	0,66330	0,90942	1,20194	1,53949	1,92279	2,35286	2,83083	3,35757	3,93335
-10,0	0,44554	0,52622	0,66990	0,87223	1,12742	1,42959	1,77592	2,16690	2,60366	3,08732	3,61876	4,19845
-12,5	0,70541	0,78682	0,93354	1,14119	1,40406	1,71545	2,07077	2,46992	2,91427	3,40480	3,94213	4,52680
-15,0	1,03217	1,11371	1,26191	1,47294	1,74124	2,06002	2,42380	2,83121	3,28355	3,78170	4,32588	4,91665
-17,5	1,42697	1,50839	1,65711	1,86997	2,14170	2,46566	2,83623	3,25129	3,71125	4,21681	4,76841	5,36643
-20,0	1,89043	1,97160	2,12035	2,33398	2,60752	2,93479	3,31048	3,73178	4,19840	4,71057	5,26915	5,87496
-22,5	2,42289	2,50383	2,65235	2,86601	3,14021	3,46937	3,84858	4,27480	4,74698	5,26525	5,83047	6,44401
-25,0	3,02482	3,10554	3,25358	3,46672	3,74088	4,07118	4,45292	4,88311	5,36035	5,88454	6,45649	7,07762
-27,5	3,69595	3,77662	3,92417	4,13680	4,41085	4,74204	5,12614	5,55995	6,04208	6,57224	7,15106	7,77987
-30,0	4,43675	4,51761	4,66491	4,87726	5,15143	5,48364	5,87017	6,30779	6,79480	7,33093	7,91672	8,55327